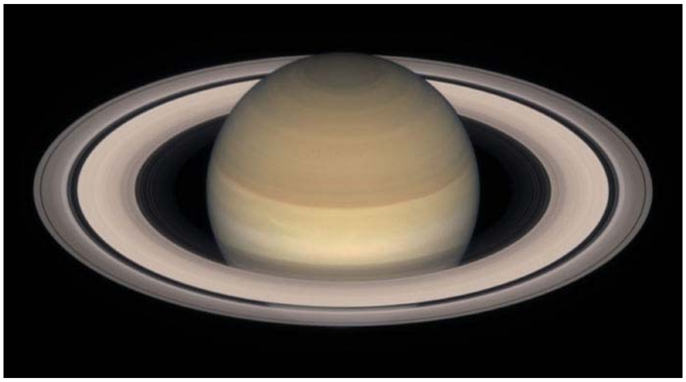
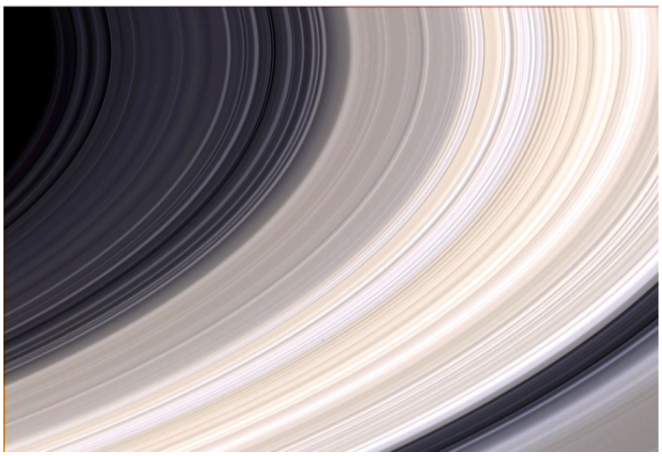
Tidal Forces - Let 'er Rip!





As the Moon orbits Earth, its gravitational pull raises the familiar tides in the ocean water, but did you know that it also raises 'earth tides' in the crust of earth? These tides are up to 50 centimeters in height and span continent-sized areas. The Earth also raises 'body tides' on the moon with a height of 5 meters!

Now imagine that the moon were so close that it could no longer hold itself together against these tidal deformations. The distance were Earth's gravity will 'tidally disrupt' a solid satellite like the moon is called the tidal radius. One of the most dramatic examples of this is the rings of Saturn, where a nearby moon was disrupted, or prevented from forming in the first place!

Images courtesy NASA/Hubble and Cassini.

Problem 1 - The location of the tidal radius (also called the Roche Limit) for two bodies is given by the formula $d = 2.4x\ R\ (\rho_M/\rho_m)^{1/3}$ where ρ_M is the density of the primary body, ρ_m is the density of the satellite, and R is the radius of the main body. For the Earth-Moon system, what is the Roche Limit if R = 6,378 km, ρ_M = 5.5 gm/cm^3 and ρ_m = 2.5 gm/cm^3? (Note, the Roche Limit, d, will be in kilometers if R is also in kilometers, and so long as the densities are in the same units.)

Problem 2 - Saturn's moons are made of ice with a density of about 1.2 gm/cm^3 .If Saturn's density is 0.7 gm/cm^3 and its radius is R = 58,000 km, how does its Roche Limit compare to the span of the ring system which extends from 66,000 km to 480,000 km from the planet's center?

Problem 3 - In searching for planets orbiting other stars, many bodies similar to Jupiter in mass have been found orbiting sun-like stars at distances of only 3 million km. What is the Roche Limit for a star like our Sun if its radius is R = 600,000 km, and the densities are ρ(planet) = 1.3 gm/cm^3 and ρ(star) = 1.5 gm/cm^3?

Answer Key

Problem 1 -
$d = 2.4xR\ (\rho_M/\rho_m)^{1/3}$
with R = 6,378 km, ρ_M = 5.5 gm/cm^3 and ρ_m = 2.5 gm/cm^3
$d = 2.4 \times 6{,}378\ (5.5/2.5)^{1/3}$
d = 19,900 km.

This distance is inside the orbits of geosynchronous communications satellites (42,164 kilometers). They are not destroyed because the tensile strength of aluminum is far higher than the gravitational tidal forces they feel.

Fortunately our moon is at a distance of 363,000 (perigee) and is steadily moving farther away by 3 centimeters per year!

Problem 2 -
$d = 2.4 \times 58{,}000\ (0.7/1.2)^{1/3}$
d = 116,900 kilometers.

The span of the ring system which extends from 66,000 km to 480,000 km from the planet's center, so the Roche Limit is inside the ring system.

Problem 3 - In searching for planets orbiting other stars, many bodies similar to Jupiter in mass have been found orbiting sun-like stars at distances of only 3 million km. What is the Roche Limit for a star like our Sun if its radius is R = 600,000 km, and the densities are ρ(planet) = 1.3 gm/cm^3 and ρ(star) = 1.5 gm/cm^3?

$d = 2.4 \times 600{,}000\ (1.5/1.3)^{1/3}$
d = 1.5 million kilometers.

So the large planets being detected are very close to the Roche Limits for their stars!